Analysis and Synthesis of Broad-Band Symmetric Power Dividing Trees

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Abstract—**A planar power divider with 2^m output ports consists of 2-way equal-power dividing sections coupled after each other to form a tree-like structure. This paper deals with the synthesis of such symmetric structures, thus forming a network that divides the incoming power into equal parts over a broad band. The analysis is done by the even and odd modes. An optimization program has been written which can optimize the total bandwidth with nearly equal-ripple response. Tables are given for synthesized power dividers with 4, 8, and 16 output ports and with a VSWR equal to 1.05, 1.1, and 1.2. The bandwidth, f_{max}/f_{min} of the power dividers in the tables is between 1.7 and 7. A 4-way divider with 7 transformers in the even mode and 3 isolating resistors in each odd mode has been built with the center frequency 5 GHz. The total bandwidth of the whole divider, which is theoretically 4.5, was measured to be 4.1.**

I. Introduction

A PLANAR POWER divider with 2^m output ports consists of 2-way Wilkinson hybrids coupled after each other to form a tree-like structure. This paper deals with the synthesis of such symmetric structures where every 2-way divider is an equal-power divider, thus forming a network that divides the incoming power into equal parts. Each port is matched and the output ports are isolated from each other. The circuit is built up by coupled and uncoupled quarter-wave transformers and isolation resistors.

The circuit is analyzed by using the even and odd modes. Thus the circuit is described by a number of excitation modes. Each mode can be described by a 2-port network. The analysis of the power divider is done by combining the results of the different modes. The synthesis of 2-way dividers has been treated in [1]–[4] (a special case of the N-way nonplanar power dividers). The 4-way planar divider has been treated to some extent in [5], while it is shown in [6] how to broad band the even mode (that is the input port). This paper discusses the synthesis of planar power dividers with 2^m output ports.

II. Analysis

Let $N=2^m$ be the number of output ports. The S-parameters are calculated by using the even- and odd-mode analysis. The analysis shows that the excitation of the input port of the power divider is described by the even mode. By symmetry it can be shown that the reflection coefficients at the output ports are equal and that the number of different isolation curves is reduced. The S-parameters at the output ports are written as an N-order vector with one reflection coefficient S_{22}, and $N-1$ isolating transmission coefficients $S_{23}-S_{2,N+1}$

$$S=\begin{bmatrix} S_{22} \\ S_{23} \\ \vdots \\ S_{2,N+1} \end{bmatrix}. \tag{1}$$

Next we wish to describe the S-parameters in (1) as functions of the reflection coefficients of the even- and odd-mode equivalent circuits. This is done by inserting N voltage sources at every output port. The sum of the N voltage sources is equal to one at one port and zero at the other ports. Thus the excitation forms an $N \times N$ matrix with the elements α_{ij} where i represents the output port number, $i=1$ being the excited port and j represents the mode number, $j=N$ being the even mode and the other j's being the odd modes.

The following must hold for the α-matrix of a binary power divider.

1) The sum of the first row ($i=1$) must be equal to one.
2) The sum of the other rows must be equal to zero.
3) The sum of the Nth column must be equal to one (even mode).
4) The sum of the other columns must be equal to zero.
5) $|\alpha_{ij}|$ must be equal for all elements, and all elements in the Nth column must have a positive sign.

A. Example

Take the 4-way divider in Fig. 1(a). Conditions 3 and 5 for $j=4$ gives

$$\alpha_{14}=\alpha_{24}=\alpha_{34}=\alpha_{44}=1/4. \tag{2}$$

Condition 1 gives

$$\alpha_{11}=\alpha_{12}=\alpha_{13}=\alpha_{14}=1/4. \tag{3}$$

Conditions 2 and 4 give 3 types of columns that can change place in the matrix, each one corresponds to a certain odd mode circuit. The α-matrix is

$$\alpha=\begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ -1/4 & -1/4 & 1/4 & 1/4 \\ -1/4 & 1/4 & -1/4 & 1/4 \\ 1/4 & -1/4 & -1/4 & 1/4 \end{bmatrix}. \tag{4}$$

Looking at the 4th column this mode has magnetic walls between the output ports. The equivalent circuit is the even mode circuit (Γ_e in Fig 1(b)). The mode in the 3rd

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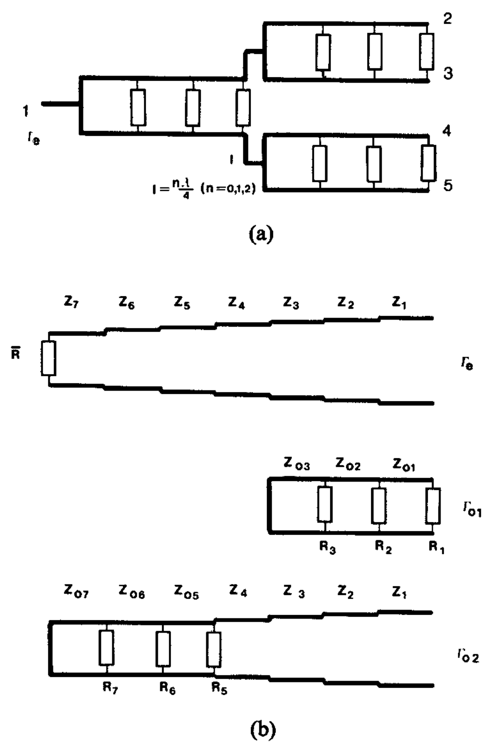


Fig. 1. (a) A 4-way power divider. (b) Equivalent networks for the even and odd modes.

column has a magnetic wall between ports 2 and 3, 4, and 5, and an electric wall between ports 3 and 4. The equivalent circuit is an odd mode with Γ_{o2}. The first and second columns have the same odd-mode equivalent circuits with Γ_{o1} (Fig 1(b)), because of the symmetry in the power divider. Thus the first and second columns can be put together. The reduced matrix is defined as the β-matrix. The β-matrix for a 4-way divider is

$$\beta=\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ -1/2 & 1/4 & 1/4 \\ 0 & -1/4 & 1/4 \\ 0 & -1/4 & 1/4 \end{bmatrix}. \tag{5}$$

The β-matrix obeys the same conditions as the α-matrix except for condition (5). The following must hold for the β-matrix: concerning the above conditions $|\beta_{ij}|$ must be equal in any column, and all elements in the Nth column must have a positive sign.

The general β-matrix for these power dividers can be written as

$$\beta=\begin{bmatrix} 1/2 & 1/4 & \cdots & 1/N & 1/N \\ -1/2 & 1/4 & \cdots & \cdot & \cdot \\ 0 & -1/4 & & \cdot & \cdot \\ 0 & -1/4 & & \cdot & \cdot \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -1/N & 1/N \end{bmatrix}. \tag{6}$$

The β-matrix for a 8-way power divider consists of one more odd-mode column compared to the 4-way divider, with 1/8 4 times and $-1/8$ 4 times and the even-mode column with 1/8 8 times. Put the number of columns

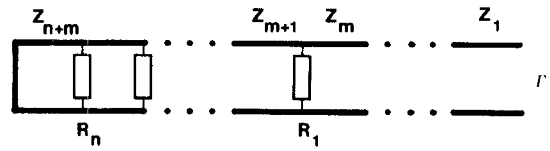


Fig. 2. The type of circuit which can be synthesized with nearly equal-ripple to the order of the number of resistances.

equal to k. Condition (1) gives

$$\sum_{i=1}^{k-1} 1/2^i + 1/N = 1. \tag{7}$$

But

$$\sum_{i=1}^{k-1} (1/2)^i = 1-(1/2)^{k-1}. \tag{8}$$

Thus

$$k = 1+\log_2 N = 1+m. \tag{9}$$

The vector equation for the S-parameters is

$$S=\beta\Gamma \tag{10}$$

where $\Gamma=[\Gamma_{o1}, \Gamma_{o2}\cdots\Gamma_e]^{-1}$.

Γ_{o1} is the reflection coefficient of the odd mode of the dividing section near the output port. Γ_{o2} is the reflection coefficient of the odd mode of the next dividing section in cascade with the even-mode circuit of the dividing section near the output ports and so on for higher order odd modes. Γ_e is the even-mode reflection coefficient. The order of the Γ-vector is $k=1+m$.

III. Synthesis

The different modes must be synthesized individually and put together to form the matching curves of the power divider. Synthesis of the even-mode network is well known [2]. The odd-mode network Γ_{o1} in Fig. 1 is known from the synthesis of 2-way power dividers, while little is known about the networks for the other odd modes. A computer program has been made which can synthesize any network of the form shown in Fig. 2. The characteristic impedances of the quarter-wave transformers must be specified from the start, for example through the synthesis of the even mode circuit. The computer program solves the problem of nearly equal-ripple matching with the isolating resistances as variables. Thus the order of the zeros in the matching curves is equal to the number of resistances independent of the number of quarter-wave transformers.

Let us put the number of resistances equal to n. The synthesis of the circuit in Fig. 2 is done by minimization of the error function E

$$E=\sum_{i=1}^{n} |\Gamma(f_i/f_0)| \tag{11}$$

where Γ is the reflection coefficient at the normalized frequency f_i/f_0. No exact solution of the equal-ripple synthesis of the circuit in Fig. 2 has been found. The choice of the normalized frequencies f_i/f_0 is critical to get the optimized bandwidth. Now, let us compare the circuit

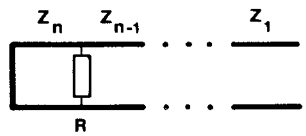
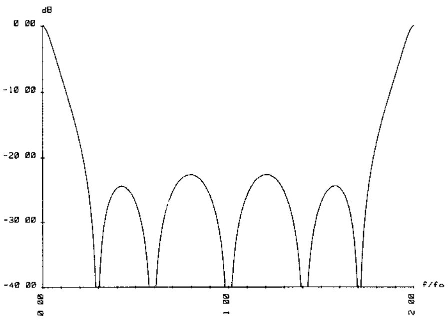


Fig. 3. The synthesis of this circuit is known and compared with the circuit in Fig. 2.



Z	R
Ω	Ω
57.2	200.6
63.15	232.6
70.7	159.1
79.2	108.9
87.4	76.83

Fig. 4. An example showing the reflection coefficient of the input of a 5th-order odd-mode circuit with a ripple level of VSWR = 1.2.

in Fig. 2 with the circuit in Fig. 3. The synthesis of this circuit is known from [7] and [8]. The characteristic impedances are here the independent variables.

Thus by specifying a lower limit of the bandwidth $f_{\min}/f_0$ the exact values of the frequencies f_i/f_0 (for the circuit in Fig. 3) are given from the equation below. The insertion loss function P_L is

$$P_L = 1 + \frac{\kappa P_n^2(x)}{1-x^2} \tag{12}$$

where P_n is an even or odd polynomial in x and κ is an arbitrary constant. In [8] it is shown that P_L has an equal ripple level if P_n is given by

$$2P_n(x) = \left(1+\sqrt{1-x_c^2}\right)T_n(x/x_c) - \left(1-\sqrt{1-x_c^2}\right)T_{n-2}(x/x_c). \tag{13}$$

T_n is the nth order Chebyshev polynomial

$$x = -\cos\frac{\pi}{2}\frac{f}{f_0}$$

$$x_c = -\cos\frac{\pi}{2}\frac{f_{\min}}{f_0}. \tag{14}$$

The zeros of P_n determine the frequencies f_i/f_0. Fig. 4 shows an example of a 5th order nearly equal-ripple matching. As can be seen the approximation is quite good.

A computer program can now be developed which optimizes the bandwidth, within a specified ripple level, for the whole power divider. The bandwidth of the power divider is defined so that every matching and isolating curve has a ripple level better than or equal to the specified ripple level. The operation of the computer program is shown in Fig 5. Input parameters are: characteristic impedances $f_{\min}/f_0$, and the ripple level for each mode and also the specified ripple level for the power divider. In the next step a first synthesis of the odd modes is done. The resulting ripple levels are compared with the specified ones. If they do not agree, the values $f_{\min}/f_0$ are changed for the next synthesis. After that the whole power divider is analyzed and the ripple levels of the matching and isolating curves within the bandwidth are compared with the specified ripple level for the whole power divider. If the values do not agree the ripple level of the odd mode with the least bandwidth is changed. The program continues until the specified ripple level, for the whole power divider is reached within ± 1 dB. The computer time on an IBM 3031 computer is less than 1 min for most cases. The Appendix shows some synthesized power dividers with VSWR = 1.2, 1.1, and 1.05 and with 4, 8, and 16 output ports.

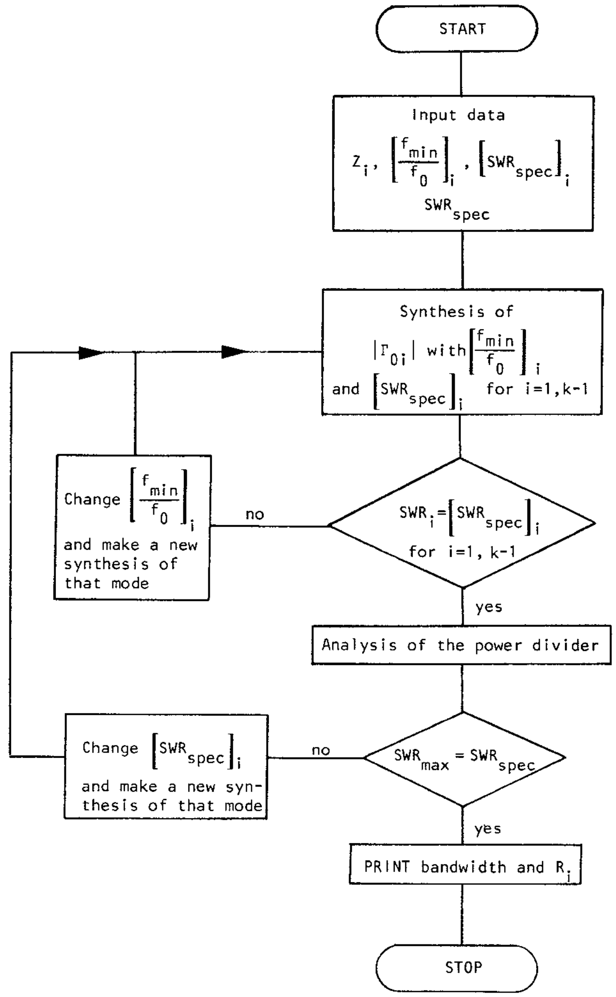


Fig. 5. The operation of the computer program where i represents the odd modes.

IV. Experimental Model and Measured Results

An experimental 4-way power divider has been built with the center frequency 5 GHz. It was built in stripline with a groundplane spacing of 3.15 mm and with RT/duroid 5870 ($\epsilon_r = 2.35$) as dielectric. The power divider consists of two different 2-way dividing section with

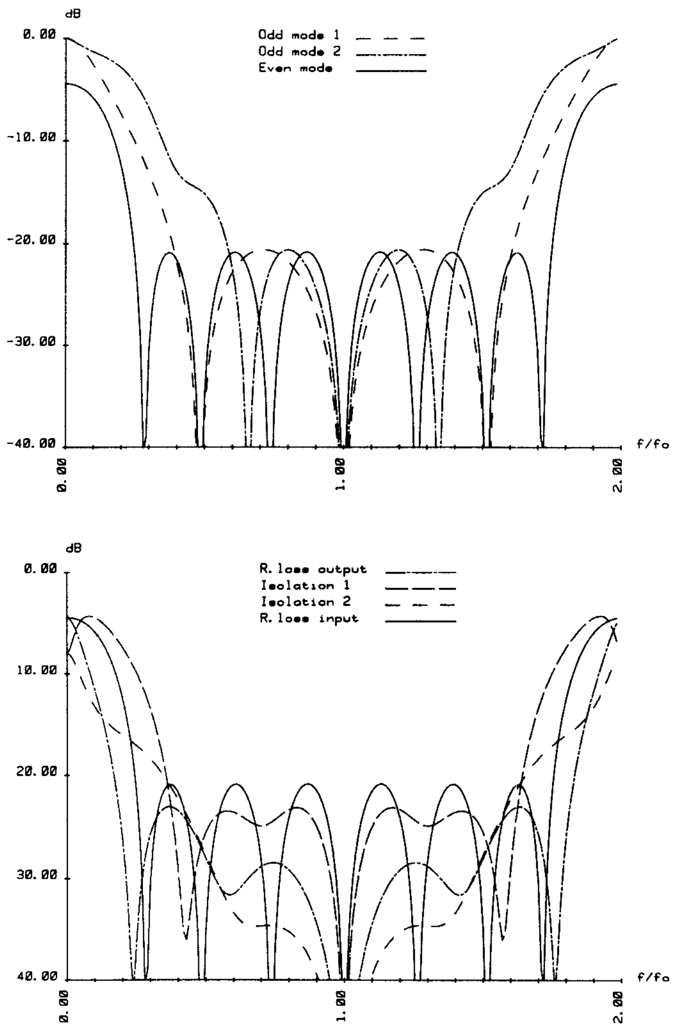


Fig. 6. The theoretical matching curves for the 4-way power divider.

TABLE I
SYNTHESIZED VALUES FOR A 4-WAY POWER DIVIDER WITH VSWR = 1.2 AND ISOLATIONS = 20.83 dB

Z_{oe} Ω	Z_{oo} Ω	Z_o Ω	R Ω
58.3	56		290
67.4	56		218
81.2	56		95
—	—	50	—
61.6	59.5		735
74.2	59.5		362
85.9	59.5		79

Note: The value of the characteristic impedances and isolating resistors are number from the output to the input.

3 quarter-wave transformers in each section and a quarter-wave transformer inbetween. This results in 7 transformers in the even mode. Table I shows the characteristic impedances for the power divider together with the synthesized values of the isolating resistances, with a VSWR equal to 1.2. Compared to the Appendix, K is equal to 3, and L is equal to 1. The bandwidth (f_{max}/f_{min}) is equal to 4.53 limited by the isolation curves. The theoretical curves are shown in Fig. 6. The measured results are shown in Table II. The VSWR of the measured matching curves were equal to 1.3 at a few points. A photo of the circuit is shown in Fig. 7.

TABLE II
BANDWIDTHS OF A 4-WAY POWER DIVIDER IN TABLE I

	Measured	Theory
Bandwidth of VSWR input	7.1	7.0
Bandwidth of VSWR output	10.2	10.1
Bandwidth of isolation	4.6	4.5 (ports 2–3,4–5)
Bandwidth of isolation	4.1	4.5 (ports 2–4,···)

V. CONCLUSIONS

The synthesis of a broad-band symmetric binary power dividers has been solved by means of computer optimization. It is shown how the symmetric binary power divider can be analyzed with the help of the even and odd modes. The result is a 2-port network for the even mode and a number of odd-mode networks. All of these networks are synthesized to get the optimum bandwidth. A 4-way power divider has been built with the center frequency 5 GHz. The theoretical bandwidth was 4.5 and the measured one 4.1.

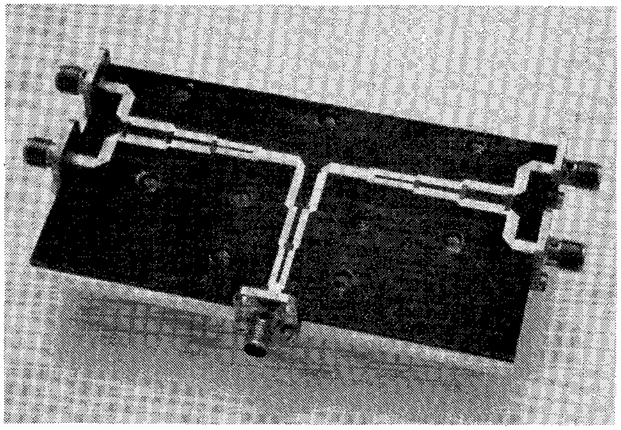


Fig. 7. Photo of the experimental stripline 4-way power divider with the top dielectric and groundplane removed.

Appendix

The tables below show some synthesized power dividers with 4, 8, and 16 output ports. K is equal to the number of quarter-wave transformers in all 2-way dividing sections and L is the number of quarter-wave transformers between the 2-way dividing sections. For example compare the 4-way divider in Fig. 7 where $K=3$ and $L=1$. B is equal to the bandwidth (f_{max}/f_{min}). All the coupled sections are assumed to be uncoupled ($Z_e = Z_o = Z$). The normalized characteristic impedance Z and the normalized isolating resistances R are numbered from the output to the input port.

Power divider 1 → 4, VSWR = 1.2

K L B: 2 0 3.08		K L B: 2 1 3.30		K L B: 2 2 3.40		K L B: 3 0 4.97		K L B: 3 1 5.06		K L B: 3 2 5.35	
Z	R	Z	R	Z	R	Z	R	Z	R	Z	R
1.24	3.69	1.20	4.1	1.18	3.78	1.18	5.67	1.16	5.54	1.15	4.73
1.68	2.51	1.51	2.17	1.41	2.31	1.41	4.36	1.35	4.37	1.31	4.51
1.19	4.85	1.0	-	0.89	-	1.77	2.60	1.62	2.53	1.53	2.92
1.61	2.83	1.33	8.07	1.13	-	1.13	4.35	1.0	-	1.82	-
		1.66	2.63	1.42	5.99	1.42	6.98	1.23	8.12	1.10	-
				1.70	3.28	1.70	5.80	1.48	5.88	1.31	17.6
								1.72	2.44	1.53	5.53
										1.73	2.54

PD 1 → 4, VSWR = 1.2

K L B: 4 0 6.2		K L B: 4 1 6.7	
Z	R	Z	R
1.15	6.52	1.15	6.90
1.31	6.97	1.27	7.01
1.53	4.37	1.46	4.26
1.82	3.15	1.70	2.79
1.10	37.26	1.0	-
1.31	10.12	1.18	-
1.53	6.38	1.37	4.08
1.73	2.30	1.57	8.15
		1.74	5.89

PD 1 → 8, VSWR = 1.2

K L B: 2 0 3.3		K L B: 3 0 5.2		K L B: 4 0 7.0	
Z	R	Z	R	Z	R
1.22	4.04	1.17	5.40	1.15	6.95
1.61	2.25	1.37	4.40	1.28	7.03
1.16	3.86	1.69	2.65	1.47	4.28
1.73	3.36	1.08	6.14	1.73	2.80
1.24	7.11	1.41	5.05	1.04	7.1
1.64	2.61	1.85	3.78	1.28	9.35
		1.19	10.26	1.57	4.55
		1.46	5.56	1.92	5.35
		1.71	2.07	1.16	11.74
				1.36	10.33
				1.57	4.56
				1.74	2.15

PD 1 → 4, VSWR = 1.1

K L B: 2 0 2.3		K L B: 2 1 2.1		K L B: 2 2 2.1		K L B: 3 0 3.8		K L B: 3 1 3.6		K L B: 3 2 3.2	
Z	R	Z	R	Z	R	Z	R	Z	R	Z	R
1.19	4.88	1.15	4.91	1.13	4.98	1.12	7.45	1.11	6.88	1.10	7.5
1.64	1.97	1.46	1.89	1.35	1.82	1.35	4.15	1.29	4.16	1.24	4.18
1.22	4.87	1.0	-	0.87	-	1.74	2.18	1.58	2.19	1.47	2.0
1.68	2.78	1.37	9.29	1.15	-	1.15	7.08	1.0	-	0.90	-
		1.74	2.57	1.48	8.72	1.48	5.51	1.27	14.43	1.11	-
				1.78	2.39	1.78	3.35	1.56	6.20	1.36	19.65
								1.80	2.26	1.61	5.5
										1.82	2.3

PD 1 → 4, VSWR = 1.1

K L B: 4 0 4.0		K L B: 4 1 5.1	
Z	R	Z	R
1.10	9.71	1.09	7.88
1.24	6.76	1.21	6.81
1.47	4.06	1.39	4.06
1.80	2.38	1.66	2.67
1.11	59.56	1.0	-
1.36	10.99	1.21	-
1.61	4.91	1.43	7.0
1.82	1.87	1.65	7.32
		1.83	3.73

PD 1 → 8, VSWR = 1.1

K L B: 2 0 2.3		K L B: 3 0 3.8		K L B: 4 0 4.8	
Z	R	Z	R	Z	R
1.16	4.97	1.11	7.73	1.09	9.71
1.52	1.90	1.29	4.15	1.20	6.88
1.13	4.56	1.60	2.03	1.38	3.97
1.77	2.44	1.05	7.89	1.64	2.23
1.31	7.38	1.41	4.41	1.01	8.44
1.73	2.47	1.91	2.75	1.26	7.64
		1.28	10.67	1.59	4.67
		1.55	5.06	1.99	3.81
		1.80	2.14	1.22	75.0
				1.45	15.9
				1.67	5.64
				1.83	2.43

PD 1 → 4, VSWR = 1.05

K L B: 2 0 1.9		K L B: 2 1 1.7		K L B: 2 2 1.8		K L B: 3 0 3.0		K L B: 3 1 2.6		K L B: 3 2 3.2	
Z	R	Z	R	Z	R	Z	R	Z	R	Z	R
1.15	5.48	1.11	5.56	1.09	5.69	1.09	9.74	1.08	9.50	1.07	8.66
1.62	1.84	1.42	1.75	1.31	1.68	1.31	4.06	1.24	4.05	1.20	4.08
1.24	5.47	1.0	-	0.86	-	1.71	1.93	1.54	1.84	1.42	1.86
1.73	2.50	1.41	9.24	1.17	-	1.17	9.70	1.0	-	0.89	-
		1.80	2.54	1.53	7.93	1.53	5.23	1.3	16.29	1.13	-
				1.83	2.82	1.83	2.68	1.61	5.94	1.40	11.85
								1.86	2.24	1.67	6.39
										1.87	2.97

PD 1 → 4, VSWR = 1.05

K L B: 4 0 3.4		K L B: 4 1 2.4	
Z	R	Z	R
1.07	13.05	1.06	13.72
1.20	6.86	1.17	6.99
1.42	3.87	1.35	3.77
1.77	2.10	1.62	1.93
1.13	36.31	1.0	-
1.40	10.96	1.23	12.11
1.67	5.39	1.49	5.21
1.87	2.19	1.72	2.60
		1.89	1.21

PD 1 → 8, VSWR = 1.05

K L B: 2 0 1.8		K L B: 3 0 2.8		K L B: 4 0 3.9	
Z	R	Z	R	Z	R
1.12	5.64	1.07	10.06	1.06	13.84
1.46	1.76	1.23	4.07	1.15	7.06
1.11	4.82	1.53	1.78	1.31	3.74
1.80	2.14	1.02	11.05	1.57	1.88
1.37	7.42	1.41	4.56	0.97	10.93
1.79	2.45	1.97	2.34	1.25	7.05
		1.31	17.5	1.61	4.61
		1.63	6.17	2.05	2.98
		1.86	2.26	1.28	48.9
				1.53	13.6
				1.74	5.6
				1.89	2.4

PD 1 → 16, VSWR = 1.2

K L B: 2 0 3.1		K L B: 3 0 4.7		K L B: 4 0 7.0	
Z	R	Z	R	Z	R
1.21	4.11	1.16	5.81	1.14	6.82
1.54	2.09	1.33	4.36	1.25	7.08
1.08	4.19	1.61	2.40	1.42	4.23
1.62	2.59	1.01	6.55	1.64	2.76
1.24	6.59	1.31	4.63	0.98	6.51
1.85	2.73	1.73	3.20	1.18	8.70
1.30	7.97	1.15	6.57	1.45	4.13
1.66	2.68	1.53	3.68	1.79	5.08
		1.98	1.89	1.11	10.32
		1.25	8.74	1.38	6.39
		1.50	2.49	1.70	2.71
		1.72	0.19	2.05	0.54
				1.22	8.60
				1.41	2.25
				1.59	0.03
				1.75	0.08

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Stopbands of the First-Order Bragg Interaction in a Parallel-Plate Waveguide Having Multiperiodic Wall Corrugations

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***Abstract*—The stopbands of the first-order Bragg interaction in a parallel-plate waveguide having multiperiodic wall undulations are investigated via the perturbation method of multiple scales. For a structure having two periods, the first-order Bragg interaction involves two as well as three coupled modes. Transition curves separating passbands from stopbands are found for all possible interactions. The effect of the multiple periodicity in the structure is found to be an increased band-width for the attenuation band as well as considerable attenuation throughout the band owing to the increased number of interactions. This is useful for the design of multichannel narrow-band microwave filters. The analysis is carried out for the first three dominant modes of the structure.**

I. Introduction

In a review paper on wave propagation in periodic structures, Elachi [1] pointed out the need for investigating the stopbands of multiperiodic structures. In this paper, a first step in this direction is undertaken by studying the case of a doubly periodic structure for the stopbands of the first-order Bragg interaction. For this purpose we consider the propagation of TM modes in a parallel-plate waveguide having perfectly conducting walls that are perturbed in the direction of propagation according to the following wall distortion functions:

$$g_l(z)=\delta\sin k_l z, \qquad \text{at the lower plate} \tag{1}$$

$$g_u(z)=\delta\alpha\sin(k_u z+\theta), \qquad \text{at the upper plate} \tag{2}$$

where k_l and k_u describe the wavenumbers of the undulations of the lower and upper plates, respectively. Equations (1) and (2) describe wall undulations per unit of the separation d of the two plates so that δ is a dimensionless small parameter, much smaller than unity, and small enough for the Rayleigh hypothesis to hold [2]. The parameter α is a constant and θ is a constant phase angle.

The problem of a parallel-plate waveguide with one periodicity in the wall distortion function was treated by Nayfeh and Asfar [3], where the analysis was done for the first-order interaction of two propagating modes coupled by the wall perturbation. The second-order interaction of two modes in a periodic circular guide was analyzed by Asfar and Nayfeh [4]. For unbounded media, Chu and Tamir [5] analyzed mode coupling for the mth order Bragg interaction, while Jaggard and Elachi [6] considered the case of multiharmonic media where the different order Bragg interactions may add destructively to cause the disappearance of a stopband.

In the cases cited above [3]–[6], each Bragg interaction corresponds to a stopband except for the case of multiharmonic media where additional stopbands may appear [6]. It is the purpose of this paper to find the stopbands that appear in the case of bounded media having boundary perturbations. As discussed in the sequel, there are two sets of stopbands in a structure having two periods: the first corresponds to the coupling of two modes, and the second corresponds to the coupling of three modes. The analysis is made via the method of multiple scales [7]–[8]. The same approach was used by Nayfeh and Kandil [9] to

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